

Exploring Points of Interest on a Function Graph

“Points of Interest” are locations on the real number line or in the coordinate plane where something mathematically noteworthy happens to a function’s graph, especially some significant change in the function’s behavior. Here is a glossary of some of the most important points of interest for the graph of a function $y = f(x)$:

Zero An input value $x = a$ such that $f(a) = 0$

Graphically, a zero is the x -coordinate of a point on the graph of $y = f(x)$, where the graph crosses or touches the x -axis.

CAUTION: The term “ x -intercept” is sometimes used to refer to a zero (i.e., just the value a where $f(a) = 0$), but x -intercept is more appropriately reserved for the ordered pair $(a, 0)$.

y -intercept The point $(0, f(0))$, where the graph of $y = f(x)$ crosses the y -axis.

CAUTION: The term “ y -intercept” is sometimes used to refer to only the y value $f(0)$, but is more appropriately reserved for the ordered pair $(0, f(0))$.

Maximum (or Minimum) An output value $f(a)$ that is the greatest (or smallest) function value for an interval of inputs containing $x = a$.

This value is sometimes called a *relative maximum* (or *minimum*) or a *local maximum* (or *minimum*), since graphically, a maximum (or minimum) is the y -coordinate of a relatively high (or low) spot on the graph.

NOTE: The term maximum (or minimum) value refers to just the output value $y = f(a)$, while the corresponding input $x = a$ is the location where that maximum (or minimum) occurs.

Intersection A point (a, b) that is simultaneously on two function graphs.

If we have $f(a) = g(a) = b$ for two functions f and g , then (a, b) is an intersection and will appear as a point where the two function graphs intersect or touch each other.

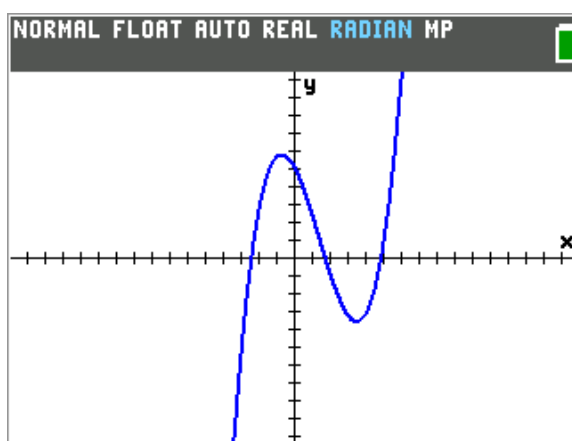
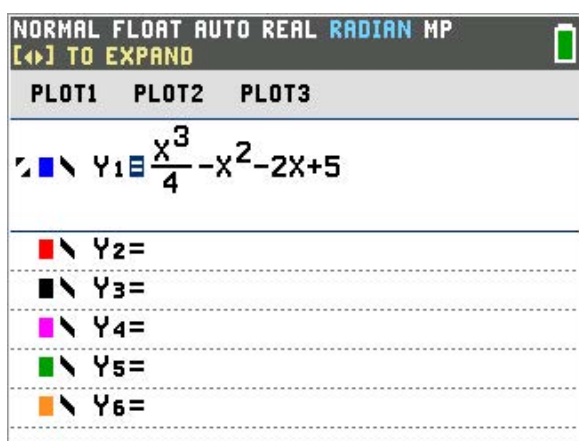
Calculus Using the TI-84 Evo

Using the “Detect POI” (Points of Interest) trace feature on the TI-84 Evo

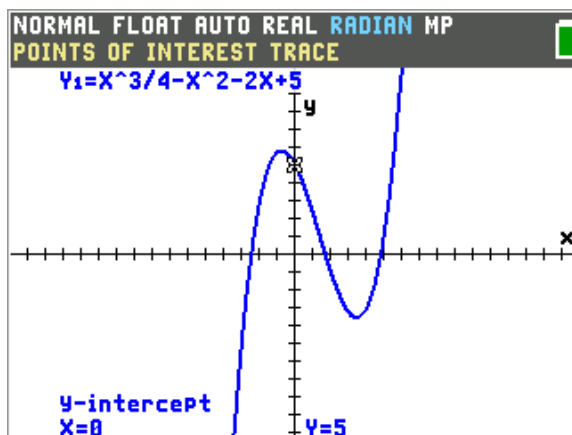
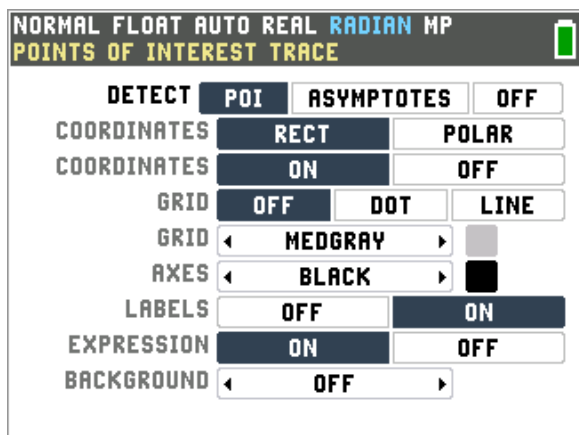
The TI-84 Evo has a function graphing format setting that will automatically detect and identify these points of interest as you trace along the graph. Here are some examples that illustrate this feature.

Example 1.

In the $\boxed{Y=}$ menu, enter the function $y = \frac{x^3}{4} - x^2 - 2x + 5$ in Y_1 and graph Y_1 in a ZDecimal window as shown here.

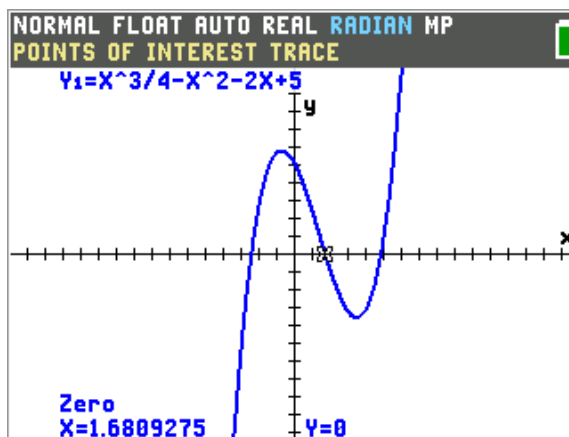
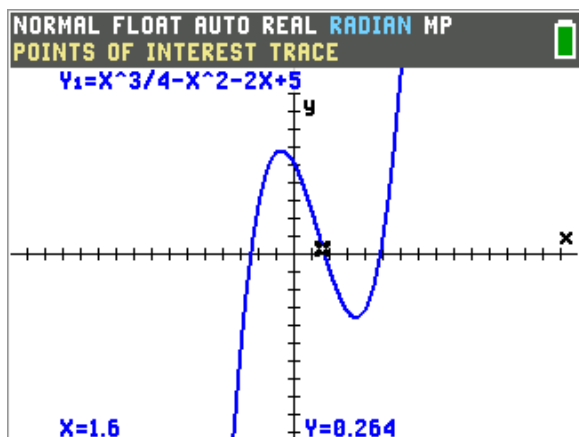


To activate the POI detection feature, go to $\boxed{format}$ ($\boxed{2nd}$ $\boxed{zoom}$) and make sure that **POI** is highlighted on the first row. Now select $\boxed{trace}$. In the ZDecimal window, the starting trace values are $x = 0$ and $y = 5$. Note that this point is identified as a **y-intercept**.

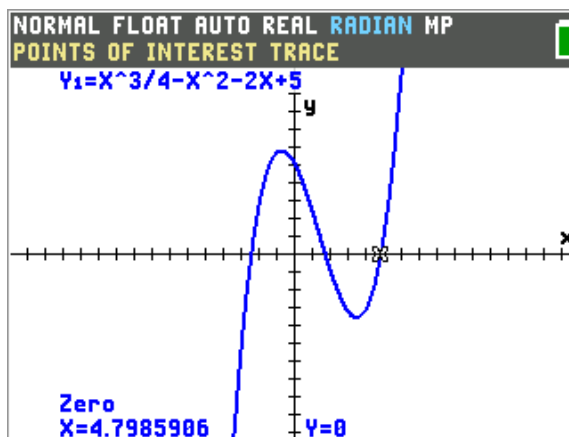
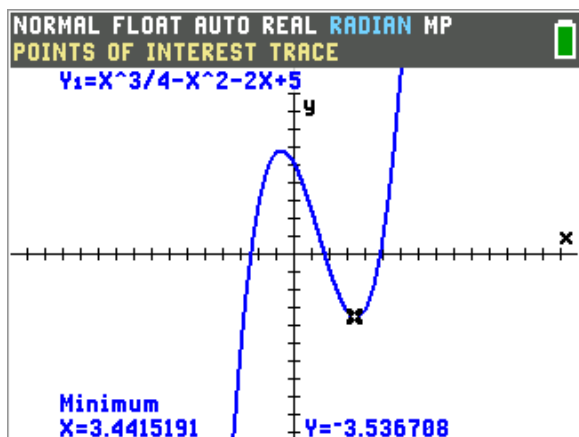


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Now trace to the right. Between the trace values $x = 1.6$ and $x = 1.7$ the **zero** $x = 1.6809275$ is detected and identified.

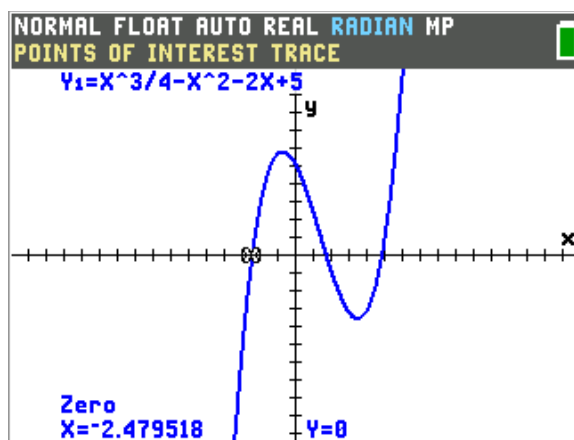
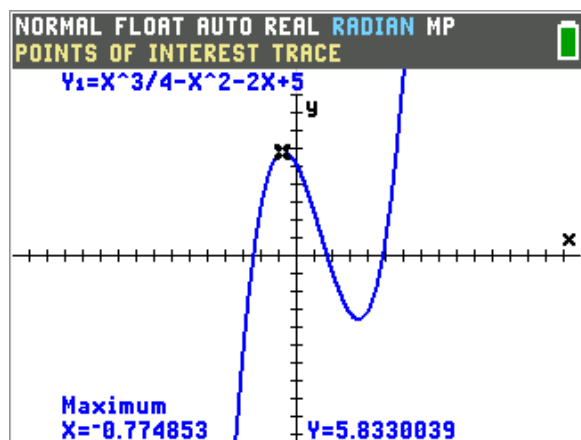


If you continue tracing to the right, the trace coordinates return to the usual decimal trace values, until the **minimum** function value is identified (between $x = 3.4$ and $x = 3.5$). Note that the $y = -3.536708$ value is the actual minimum, while the $x = 3.4415191$ value is the input where that minimum occurs. Continue tracing to the right to see another **zero** $x = 4.7985906$ identified.



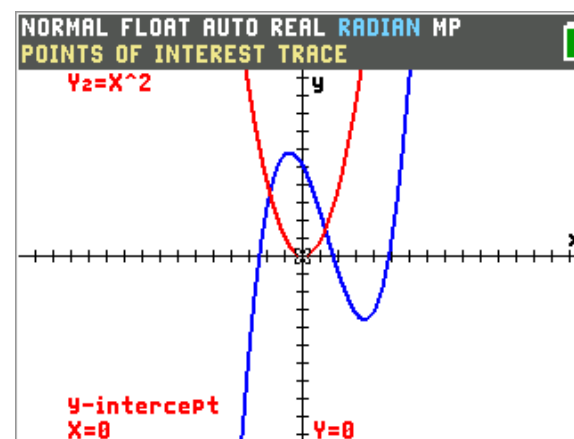
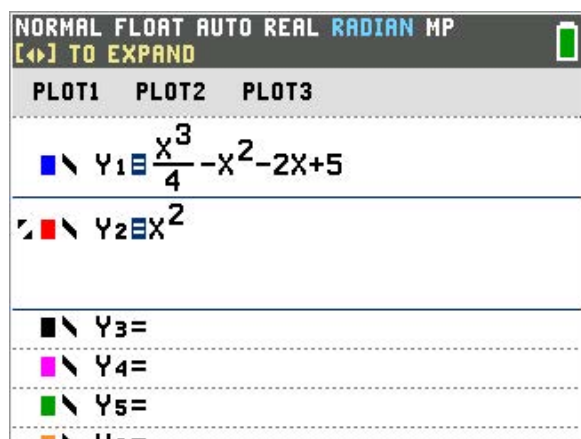
Now trace back to the left, past $x = 0$ into negative x trace values to see the **maximum** function value $y = 5.8330039$ (at $x = -0.774853$). Keep tracing left to see the **zero** $x = -2.479518$ identified.

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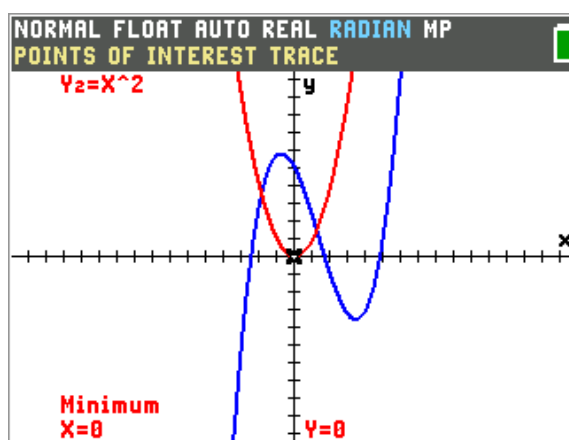
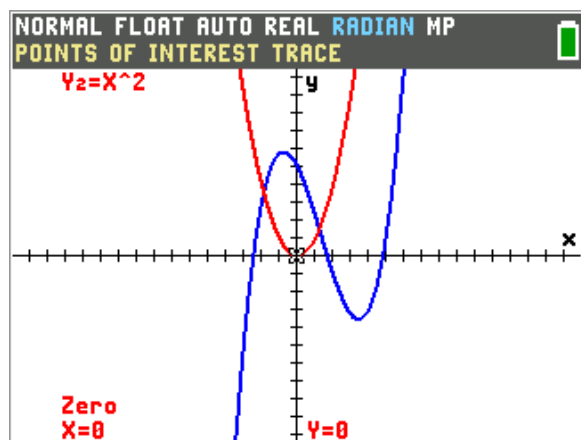
Example 2.

Let's add a simple second function $y = x^2$ in Y_2 . Go to **trace** and then up (or down) arrow to switch the trace cursor to the Y_2 graph at $x = 0$ and $y = 0$.

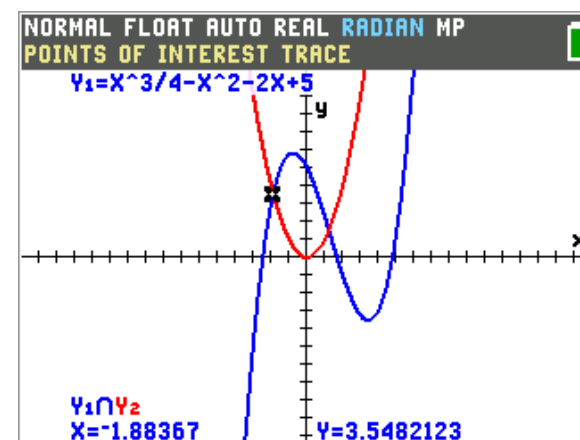
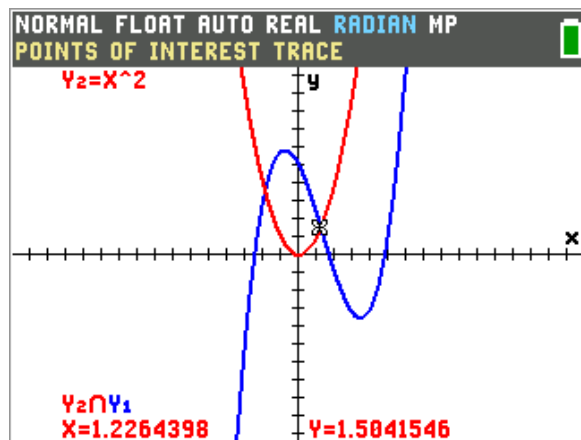


As expected, since $x = 0$, the point $(0,0)$ is identified as a **y-intercept**. Press the right arrow once and the trace cursor does not move, because $x = 0$ is also identified as a **zero** of Y_2 . Press right arrow once again and note that $y = 0$ is also a **minimum** function value located at $x = 0$.

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Now trace to the right until an **intersection** is identified (and indicated onscreen as $Y_2 \cap Y_1$) at $x = 1.2264390$, $y = 1.5041546$. Press the up or down arrow to switch the cursor to Y_1 and trace to the same intersection point (and notice that it is now indicated by $Y_1 \cap Y_2$). Trace back to the left until you find the other intersection point with $x = -1.88367$ and $y = 3.5482123$ as coordinates.



A calculus activity using the TI-84 Evo Points of Interest (POI) trace feature

Problem 1.

In calculus, the location of a point of interest for a function's derivative may be related to different kind of point of interest for the original function.

Critical values

The zeroes of the derivative f' of a function are called *critical values* for the original function f .

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- a) In the $\boxed{Y=}$ menu enter the function $y = \frac{x^5}{5} + \frac{3x^4}{8} - \frac{7x^3}{6} - \frac{3x^2}{4} - \frac{5x}{2} + 1$ in Y_1 and its numerical derivative in Y_2 (by placing the cursor in Y_2 and selecting $\boxed{\text{math}} \boxed{8}$ and completing the template as such: $\frac{d}{dx}(Y_1)|x = x$). With the POI trace feature activated, graph both functions in this window $[-4, 4] \times [-6, 6]$. Trace to each zero of the derivative (Y_2), then switch the trace to Y_1 . Which zeroes of Y_2 are also the location of a minimum of Y_1 ? a maximum? Neither?

Points of Inflection

A point $(a, f(a))$ is a point of inflection for the graph of $y = f(x)$ if the graph changes concavity at that point (from concave up to concave down, or vice-versa). In the language of calculus, a point of inflection occurs where the second derivative $f''(x)$ changes sign from positive (concave up) to negative (concave down) or vice-versa.

- b) In the $\boxed{Y=}$ menu enter the numerical derivative of Y_2 in Y_3 (this is the second numerical derivative of Y_1) and then graph all three functions. Trace to each minimum or maximum of the derivative (Y_2). Which of these are located at an inflection point for the graph of Y_1 ?

Problem 2.

In the $\boxed{Y=}$ menu enter $Y_1 = \sin(2x)$, keeping Y_2 and Y_3 as the first and second derivatives of Y_1 . Repeat parts a) and b) of Problem 1 for the function $y = \sin(2x)$.

Problem 3.

In the $\boxed{Y=}$ menu enter $Y_1 = (X - 1)^{(5/3)} - 2X$ keeping Y_2 and Y_3 as the first and second derivatives of Y_1 . Repeat parts a) and b) of Problem 1.

Problem 4.

Consider these questions.

- a) True or false? If $f'(a) = 0$, then $f(a)$ must be either a maximum or minimum value for f .
If false, find a counterexample.
- b) True or false? If $f''(a) = 0$, then $(a, f(a))$ must be an inflection point for $y = f(x)$.
If false, find a counterexample.